Invariant solutions of supersymmetric nonlinear wave equations

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Abstract:

In this talk, we present a group-theoretical symmetry analysis for the supersymmetric versions of specific nonlinear equations. Specifically, we consider supersymmetric extensions of the $(1+1)$-dimensional sine-Gordon equation, the $(1+1)$-dimensional sinh-Gordon equation and the following generalized polynomial form of the Klein-Gordon equation

$$u_{xt} + au + bu^3 + cu^5 = 0. \tag{1}$$

In each case, the supersymmetric version of the equation is constructed on the 4-dimensional Grassmannian superspace $\{(x, t, \theta_1, \theta_2)\}$. Here, the variables x and t represent the bosonic coordinates of 2-dimensional Minkowski space, while the quantities θ_1 and θ_2 are anticommuting fermionic variables. The bosonic function $u(x,t)$ is replaced by the scalar bosonic superfield

$$\Phi\,(x, t, \theta_1, \theta_2) = \frac{1}{2}u(x,t) + \theta_1\phi(x,t) + \theta_2\psi(x,t) + \theta_1\theta_2 F(x,t), \tag{2}$$

where ϕ and ψ are fermionic-valued fields and F is a bosonic field. The supersymmetric extension is constructed in such a way that it is invariant under a set of supersymmetry transformations (generated by vector fields Q_x and Q_t) which link the bosonic independent variables x and t to the fermionic independent variables θ_1 and θ_2 respectively. This is ensured by writing the supersymmetric equation in terms of covariant derivative operators D_x and D_t which anticommute with the supersymmetry generators Q_x and Q_t respectively. For each of the supersymmetric equations under consideration, we use a generalization of the method of prolongation in order to determine the Lie superalgebra of symmetries of the equation, and we present a systematic classification of all one-dimensional subalgebras of this resulting Lie superalgebra. The method of symmetry reduction then allows us to derive invariant solutions of the supersymmetric model. Some interpretation of the obtained results is given.

References:

1. A. M. Grundland, A J. Hariton and L. Snobl, J. Phys. A: Math. Theor. 42, 335203 (2009).
2. A. M. Grundland, A J. Hariton and L. Snobl, “Invariant solutions of supersymmetric nonlinear wave equations“, in preparation (2010).